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Applications and Limitations of Game Theory

Abstract

In this paper technologies of Game theory have been used to solve various types of competitive or conflicting situations and explain how the techniques of game theory are useful in solving problems related to business world mainly bidding problems. Among all these techniques, first maxima-minima Principle is used to search if a saddle point exists or not in a game i.e. to say that game is strictly determined or not. But there are games where saddle point does not exist, such type of games are solved using the Principle of dominance. The Principle of Dominance also has some limitations. It fails to solve m x 2 or 2 x n games, which can then be solved by graphical methods and other type of games can be solved by iterative method. This iterative method does not provide us an accurate solution of the game, distinctive several iterations, where as the algebraic method provide an exact solution of the game. This method can be used to solve game of any size accurately. Through comparative study of various methods of games it is concluded that Algebraic method is best available tool to solve such type of game.

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Introduction

Game is defined as an activity between two are more persons involving activities by each person according to a set of rule, at the end off which each person receives some benefit or satisfaction or suffers loss (Negative benefit). The set of rule define the game. Going through the set of rules once by the participants defines a play.

The theory of games started in the 20th century. The mathematical treatment of games was developed, when Jone Von Neumann and Morgenstern published with their work "Theory of game and Economic behaviour" in 1944. The approach to competitive problems developed, by J. Von Neumann (Known as father of game theory) utilizes the minimax principle which involve the fundamental idea of minimization of the maximum loss or the maximization of the minimum gain. The game theory is capable of analysing very simple competitive situation is cannot handle all the competitive situations that may arise.

John von Neumann

The Basics of Game Theory

Game theory turned attention away from steady-state equilibrium toward the market process. In business, game theory is beneficial for modeling competing behaviors between economic agents. Businesses often have several strategic choices that affect their ability to realize economic gain. For example, businesses may face dilemmas such as whether to retire existing products or develop new ones, lower prices relative to the competition, or employ new marketing strategies. Economists often use game theory to understand oligopoly firm behavior. It helps to predict likely outcomes when firms engage in certain behaviors, such as price-fixing and collusion.

Characteristics of Game Theory

Chance of Strategy

If in a game activities are determined by skill, it is called game of Strategy. If activities are determined by chance it is a game of chance. A game may involve game of strategy as well as game of chance.

Number of Persons

A game is called an n-person game if the number of persons playing is n.

Number of Activities

These may be finite or infinite.

Number of Alternatives Available

To each person in a particular activity is finite or infinite. A finite game has finite number of activities and an infinite game has infinite number of activities.

Pay off

A quantitative measure of satisfaction of a person gets at the end of each play is called pay-off. It is a real valued function of variable in the game. Let v_i be the pay-off of the player $P_{i;\ 1<=i<=}n$ in an n person game If $\Sigma\ v_{i\ =0}$,where i=1,2,3.....n, is called zero-sum game.

Review of Literature

In 1950, the first mathematical discussion of the prisoner's dilemma appeared, and an experiment was undertaken by notable mathematicians Merrill M. Flood and Melvin Dresher, as part of the RAND Corporation's investigations into game theory. RAND pursued the studies because of possible applications to global nuclear strategy. Around this same time, John Nash developed a criterion for mutual consistency of players' strategies, known as Nash equilibrium, applicable to a wider variety of games than the criterion proposed by von Neumann and Morgenstern. Nash proved that every finite n-player, non-zero-sum (not just 2-player zero-sum) non-cooperative game has what is now known as a Nash equilibrium in mixed strategies.

Game theory experienced a flurry of activity in the 1950s, during which time the concepts of the core, the extensive form game, fictitious play, repeated games, and the Shapley value were developed. In addition, the first applications of game theory to philosophy and science occurred during this time.

In 1979 Robert Axelrod tried setting up computer programs as players and found that in tournaments between them the winner was often a simple "tit-for-tat" program that cooperates on the first step, then on subsequent steps just does whatever its opponent did on the previous step. The same winner was also often obtained by natural selection; a fact widely taken to explain cooperation phenomena in evolutionary biology and the social sciences.

Basic Definitions

A Competitive Game

Whenever there is a competitive situation it is called a competitive game. If there are two persons involved it is called two-person game. If number of persons are more than two it is called n-person game.

Zero Sum and Non Zero-Sum Games

If an n-person game is played by n players P_1, P_2, P_3......... P_n , whose respective pay off at the end of game are v_1, v_2, v_3............. v_n respectively then the game is called zero sum game if $\Sigma\ v_{i\ =0}$,where i=1,2,3.....n at each play of the game. A game which is not zero sum is called non zero sum game.

Strategy

Strategy for a given player is a set of rules that specifies which of the available course of action he should make at each play. Strategies are of two types:

Pure Strategy

It is completely deterministic situation so pure strategy is a decision rule always to select a particular course of action.

Mixed Strategy

It is a probabilistic situation i.e.; mixed strategy is a selection among pure strategy with fix probabilities.

Two People, Zero Sum Game

A game with only two players is called a two person zero sum game and losses of one player are equal to gains of other.

Pay Off Matix

Pay off matix is matrix is matrix in which each entry represent gain or loss to one player or other player at the end of each activity by player involved in the game.

The player A's pay off matrix

Player B

$$
\text{Player A}\quad
\begin{array}{c|cccc|}
 & 1\;2\;3\;4 & \cdots & & n \\
\hline
1 & V_{11}\; V_{12}\; V_{13} & & & V_{1n} \\
2 & V_{21}\; V_{22}\; V_{23} & & & V_{2n} \\
3 & & & & \\
4 & & & & \\
m & & & & \\
 & V_{m1}\; V_{m2}\; V_{m3} & & & V_{mn} \\
\hline
\end{array}
$$

The player B's pay off matrix

Player B

$$
\text{Player A}\quad
\begin{array}{c|cccc|}
 & 1\;2\;3\;4 & \cdots & & n \\
\hline
1 & -V_{11}\; -V_{12}\; -V_{13} & & & -V_{1n} \\
2 & -V_{21}\; -V_{22}\; -V_{23} & & & -V_{2n} \\
3 & & & & \\
m & & & & \\
 & -V_{m1}\; -V_{m2}\; -V_{m3} & & & \\
\hline
\end{array}
$$

Saddle Point

A Saddle point of a Pay off Matrix is the position of an element in the pay off matrix which is minimum in its row and maximum in its column.

In the pay off matix [v_{ij}] if $\max_i\{\min(v_{ij})\} = \min_j\{\max_i (v_{ij})\} = v_{rs}$ (say)

Then pay off matrix is said to have a saddle point (r,s)

A game is said to be strictly determined if v=v=v

If a pay off matrix [v_{ij}] has a saddle point at (r,s) then the players A & B are said to have rth and sth optimal strategies and v_{rs} is called the value of the game. A game is called fair if value of game is zero i.e. $v_{rs} = 0$

If the game is without saddle point then we use Principle of dominance to reduce the size of game.

Principle of Dominance

Principle of Dominance says that if one pure strategy of a player is better or superior than another one the inferior strategy may be ignored by assigning a zero probability while searching for optimal strategies.

Dominance Rule

1. If all the entries of the ith row of the pay off matrix of the game is greater than or equal to corresponding element of the jth row, then we say that ith row is dominated to jth row and jth row is left out.
2. If all the entries of rth column of the pay off matrix of the game is less than or equal to the sth column of pay off matrix. then we say that sth column is dominated by rth column and sth column is left out.
3. If a row or column is inferior or superior to convex combination of some other rows (columns) than that row or column is left out.

Fundamental theorem of games:

Statement

Every 2 x 2 ; 2 person zero-sum game has a solution

Case-1

When the game is with saddle point, then use maxmin or minmax principle to solve the problem.

Case-2

When the game is without saddle point then we reduce the game by applying dominance rule to 2 x 2 game and solve it. For any zero sum two person game where optimal strategies are not pure strategies(i.e. there is no saddle point) and for which, the player A's payoff matrix is:

$$\text{Player A} \quad \begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \text{Player B} \\ \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \end{array}$$

$$A1 = \frac{a22-a21}{(a11+a22)-(a12+a21)}$$

$$B1 = \frac{a22-a12}{(a11+a22)-(a12+a21)}$$

$$A2 = \frac{a11-a12}{(a11+a22)-(a12+a21)}$$

$$B2 = \frac{a11-a21}{(a11+a22)-(a12+a21)}$$

$$V = \frac{a11.a22-a12.a21}{(a11+a22)-(a12+a21)}$$

Case-3

When the game is without saddle point and it is not possible to reduce it to 2 x 2 games, then we use Simplex method by transforming the problem to LPP. This method is best method out of all the methods and the result obtained is more accurate.

Hypothesis

H0

The payments should be made in accordance with the selection of strategies

H1

The payments should not be made in accordance with the non selection of strategies

Research Methodology with Illustration

Player A can choose his strategies from A1, A_2, A_3 only while player B can choose from B_1, B_2 only. The rules of game state that the payments should be made in accordance with the selection of strategies:

Strategy pair selected	Payments to be made	
A_1 B_1	A to B	Rs. 1
A_1 B_2	B to A	Rs. 6
A_2 B_1	B to A	Rs. 2
A_2 B_2	B to A	Rs. 4
A_3 B_1	A to B	Rs. 2
A_3 B_2	A to B	Rs. 6

What strategies should A and B plays in order to get the optimal benefit of the play?

We can display the payment in the following payoff matrix for player A.

		Player B B_1	B_2	Row Minimum
Player A	A_1	—1	6	—1
	A_2	2	4	2→Maxmin
	A_3	—2	—6	—6
Column Maximum		2 ↓ Minimax	6	

The matrix has a saddle point at position A_2, B_1. The optimal solution to the game is:

1. Best strategy for player A is A_2
2. Best strategy for player B is B_1 and
3. The value of the game is Rs. 2 for player A and Rs. —2 for player B.

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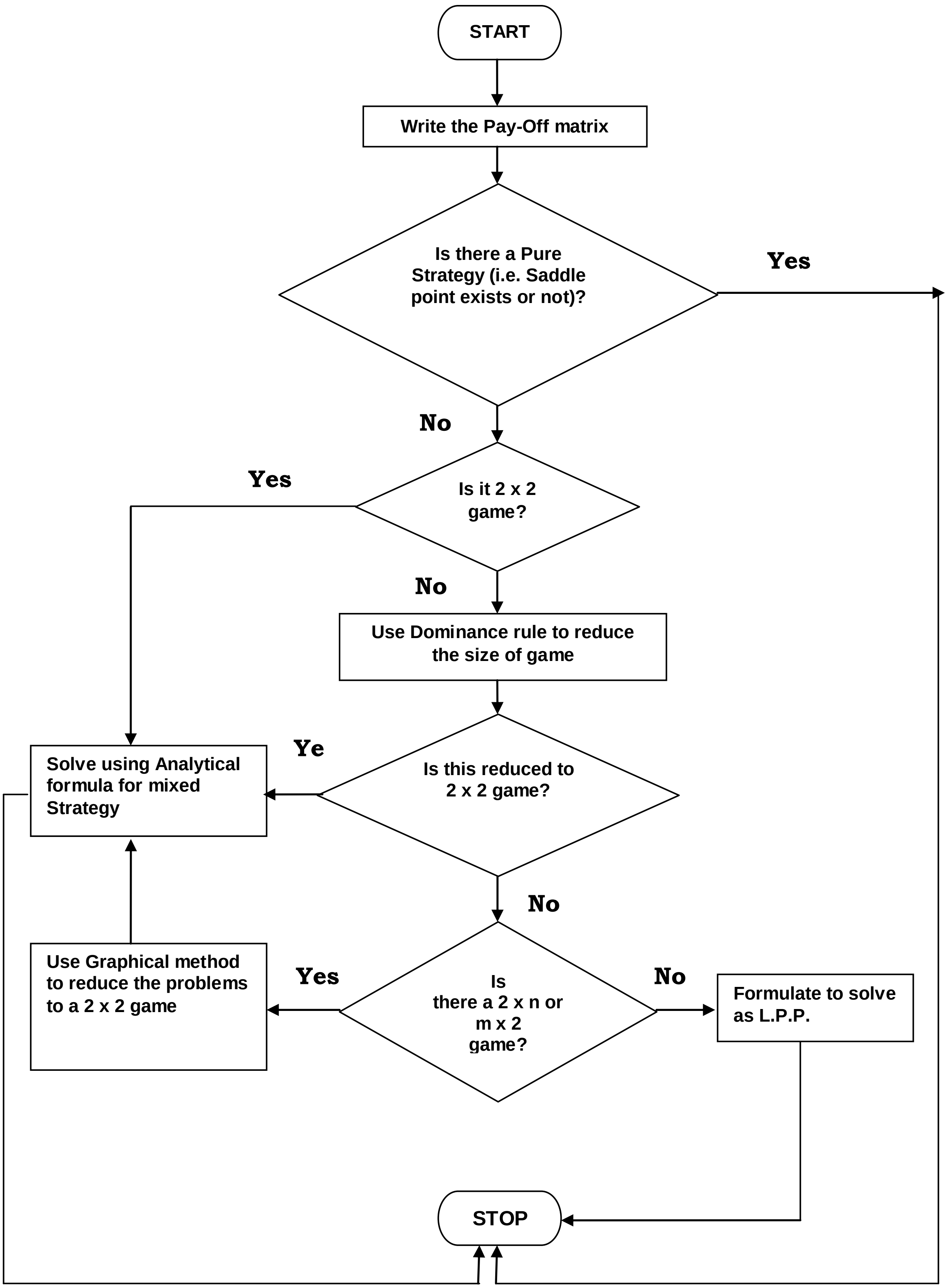
Flowchart for Systematic Application



Limitations

1. As the number of players increases in the actual business the game theory becomes more difficult.
2. It simply provides a general rule of logic not the winning strategy.
3. There is much uncertainty in actual field of business which cannot be considered in game theory.
4. Businessmen do not have adequate knowledge for the game theory.

Conclusion

After applying all the aspects of the game theory which include it limitations or say drawbacks we have come to conclusion that despite its limitation, game theory has played a major role in solving many competetive or conflicting situations. we cannot discard the role of game theory only because of its certain limitations. Since with these limitations, we are getting results by using priciple of game theory which is very close to exact result.

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